TECHNISCHE UNIVERSITÄT DARMSTADT

Bethe–Salpeter studies of mesons beyond rainbow-ladder

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College of William and Mary, Williamsburg, Virginia

Introduction

Quantum Chromodynamics is a beast

- DOF are the quarks and gluons
- non-Abelian gauge theory

$$Z[J,\eta,\bar{\eta}] = \int \mathcal{D}[A,\psi,\bar{\psi}] \exp\left\{ -\int_x \bar{\psi}(\not{D}+m)\psi + \frac{1}{4}F_{\mu\nu}^2 + \int_x A_\mu^a J_\mu^a + \bar{\eta}\psi + \eta\bar{\psi} \right\}$$

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Features various phenomena

- Asymptotic freedom
  - weakly coupled at large momenta
  - perturbation theory
- Confinement
  - Strong coupling at low momenta
  - Physical observables colourless bound-states
- Dynamical Chiral Symmetry Breaking

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⋆ Interested in: via Dyson-Schwinger equations

- Infrared properties
- Hadronic contributions to physical processes
- Determine properties of Hadronic bound-states

Introduction

Hadronic Applications

- $g-2$ (Fischer)
- bound-state masses
- electromagnetic form-factors, charge radius, transition matrix elements (Tandy)
- leptonic decay constants

Consequences of a dressed quark-gluon vertex

- $U_A(1)$ anomaly
- analytic structure of the quark propagator
- unquenching effects
- ...

Considerations

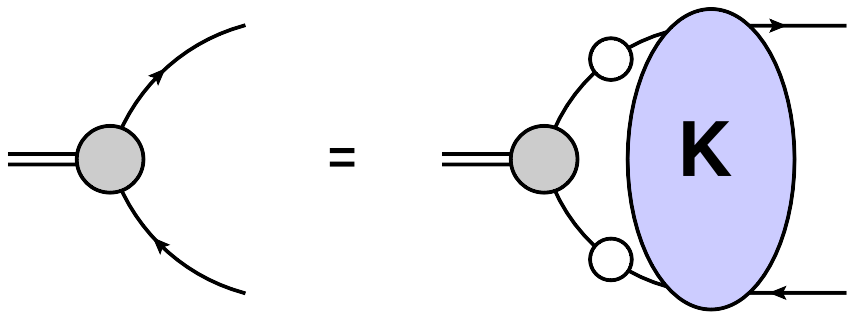
- Preservation of symmetries (pions as Goldstone bosons)
- How to make systematic improvements

Bound-state equations

Homogeneous Bethe-Salpeter equation

Consider poles in four-quark scattering amplitude

- gauge-dependent Green's fns. $\longleftrightarrow$ to physical observables
- colourless bound-states in terms of quarks and gluons

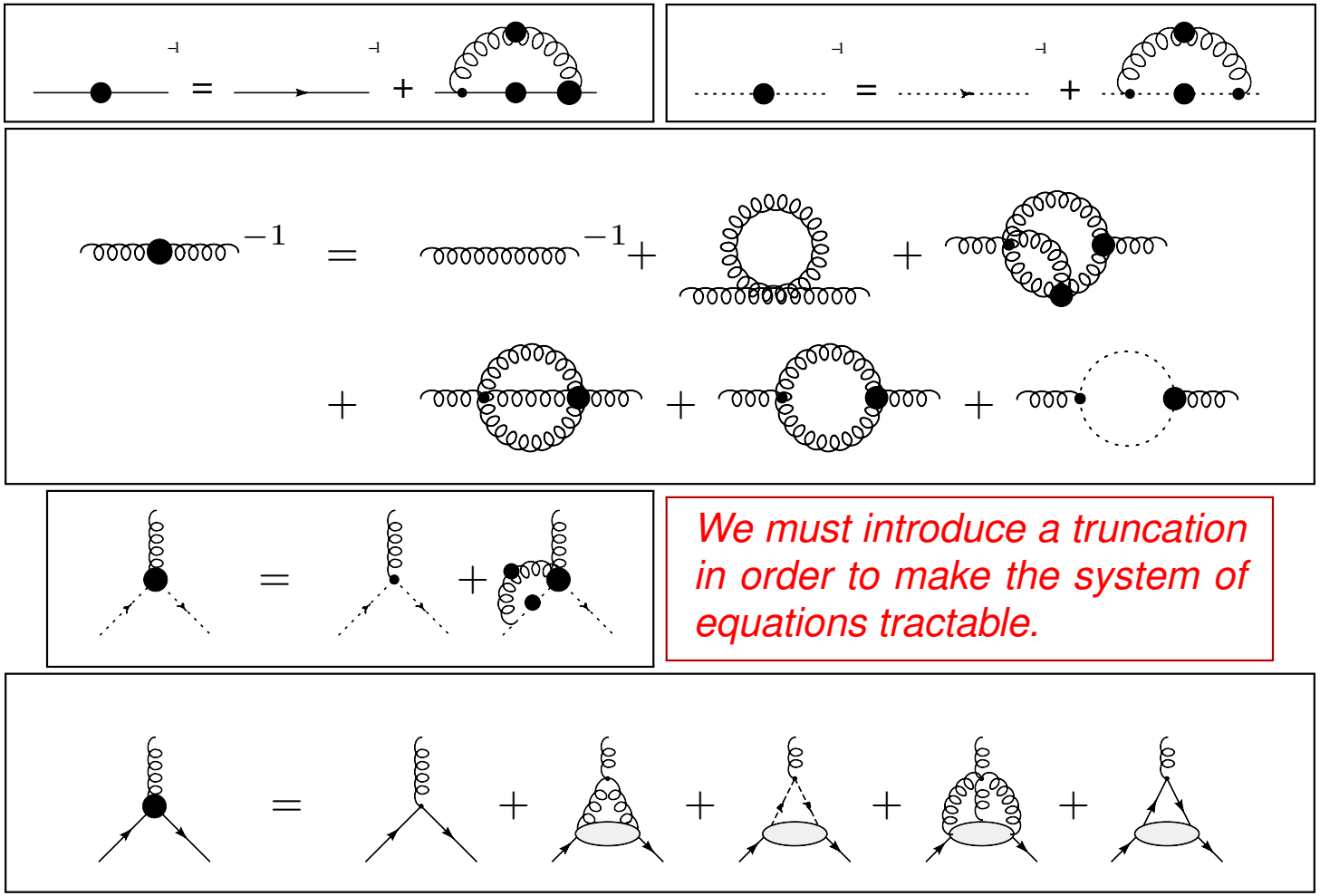


Solution requires input

- Covariant structure of amplitude (J^{PC} of bound-state)
- Quark propagator (constituent of b.s)
- Bethe-Salpeter kernel (interaction)

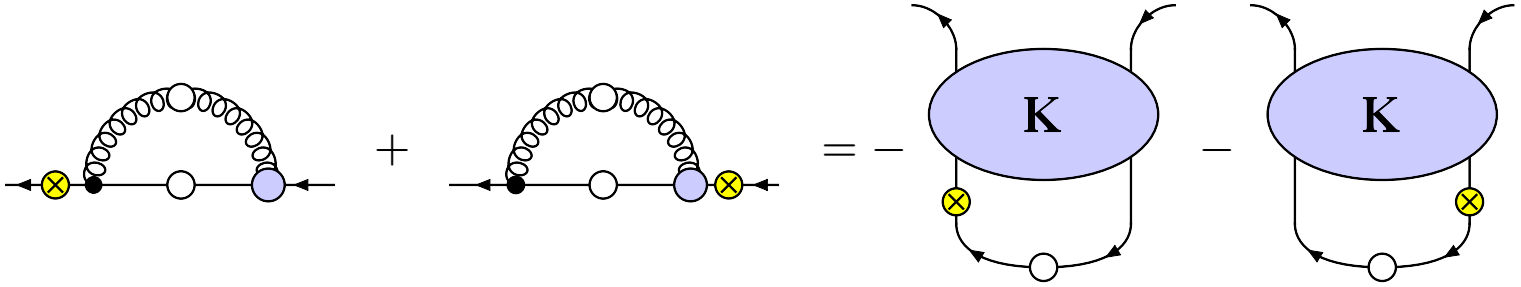
Dyson-Schwinger equations

- Exact equations of QCD.
- All propagators (inside loops) are fully dressed.



We must introduce a truncation in order to make the system of equations tractable.

Axial-vector Ward-Takahashi identity



$$\left\{\gamma^5\Sigma\left(-p_-\right)+\Sigma\left(p_+\right)\gamma^5\right\}_{\alpha\beta}=-\int K_{\alpha\gamma,\delta\beta}\left(p,q,P\right)\left\{\gamma^5 S\left(-q_-\right)+S\left(q_+\right)\gamma^5\right\}_{\gamma\delta}$$

axWTI connects quark self-energy and the Bethe-Salpeter kernel

- *truncation must consider this*
- *important for chiral properties of pseudoscalars*
  - Gell-Mann–Oakes-Renner / massless pion in the chiral limit

→ difficult and intricate to implement in practice.

$$m_\pi^2 f_\pi \simeq 2m_q\left\langle\bar{q}q\right\rangle$$

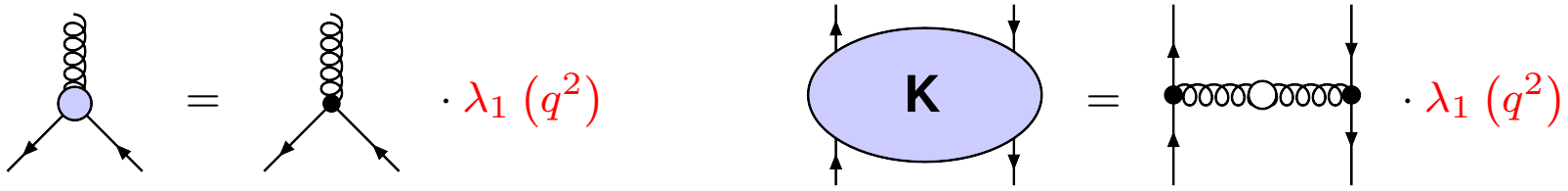
Rainbow-ladder truncation

$$\Gamma^{\mu}(k,p;\mu) = iZ_2\gamma^{\mu}$$

$$K_{\alpha\gamma,\beta\delta}(p,q,P) = Z_2^2 g^2 \left(\frac{\lambda^i}{2}\right)_{AC} \left(\frac{\lambda^i}{2}\right)_{BD} (i\gamma^{\mu})_{\alpha\gamma} D^{\mu\nu}(q) (i\gamma^{\nu})_{\beta\delta}$$

- Iterated one-gluon exchange

Rainbow-ladder truncation



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- Iterated one-gluon exchange
- Also: dress γ^μ vertex with function $\lambda_1\left(q^2\right)$
- Reduces twelve components of k^2, p^2, q^2 to one with only q^2 dependence.

Combination $\lambda_1\left(q^2\right) D^{\mu\nu}\left(q\right)$ *usually modelled as an effective interaction*

Rainbow-ladder truncation

Successes

- Largely depend on generating DCSB consistent with AXWTI.
- Get it right $\longrightarrow$ correct (light) pseudoscalar observables

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- Quantities tend to be insensitive to the model details.

Cannot learn about details of interaction from meson observables.

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Philosophical improvements

- Use calculated Lattice/DSE/FRG gluon
- Model quark-gluon vertex dressing
- Model quark-mass dependence
- *use solutions of quark-gluon vertex DSE as a starting point*

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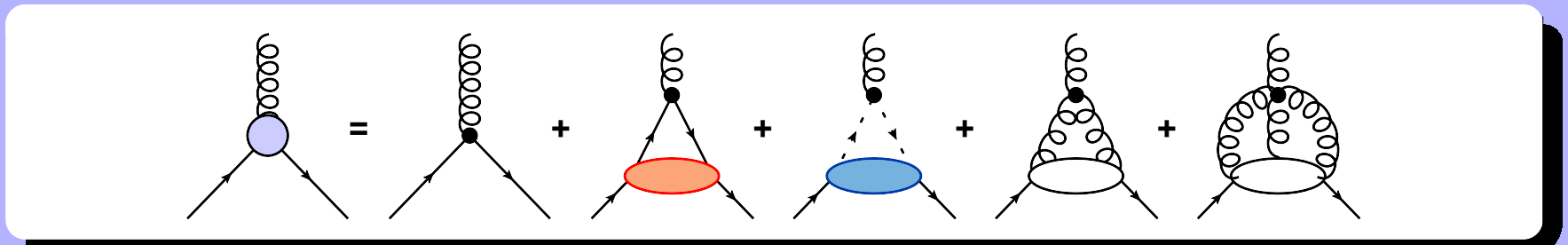
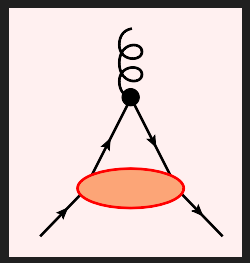
What about considering the Quark-Gluon Vertex DSE?

Quark-Gluon Vertex



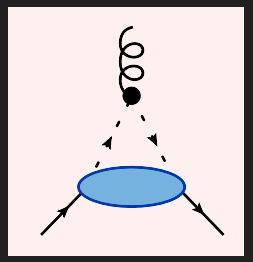
Hadronic Contributions



- Abelian-like corrections.
- resonant contributions.

Unquenching Effects

Dominant Yang-Mills part

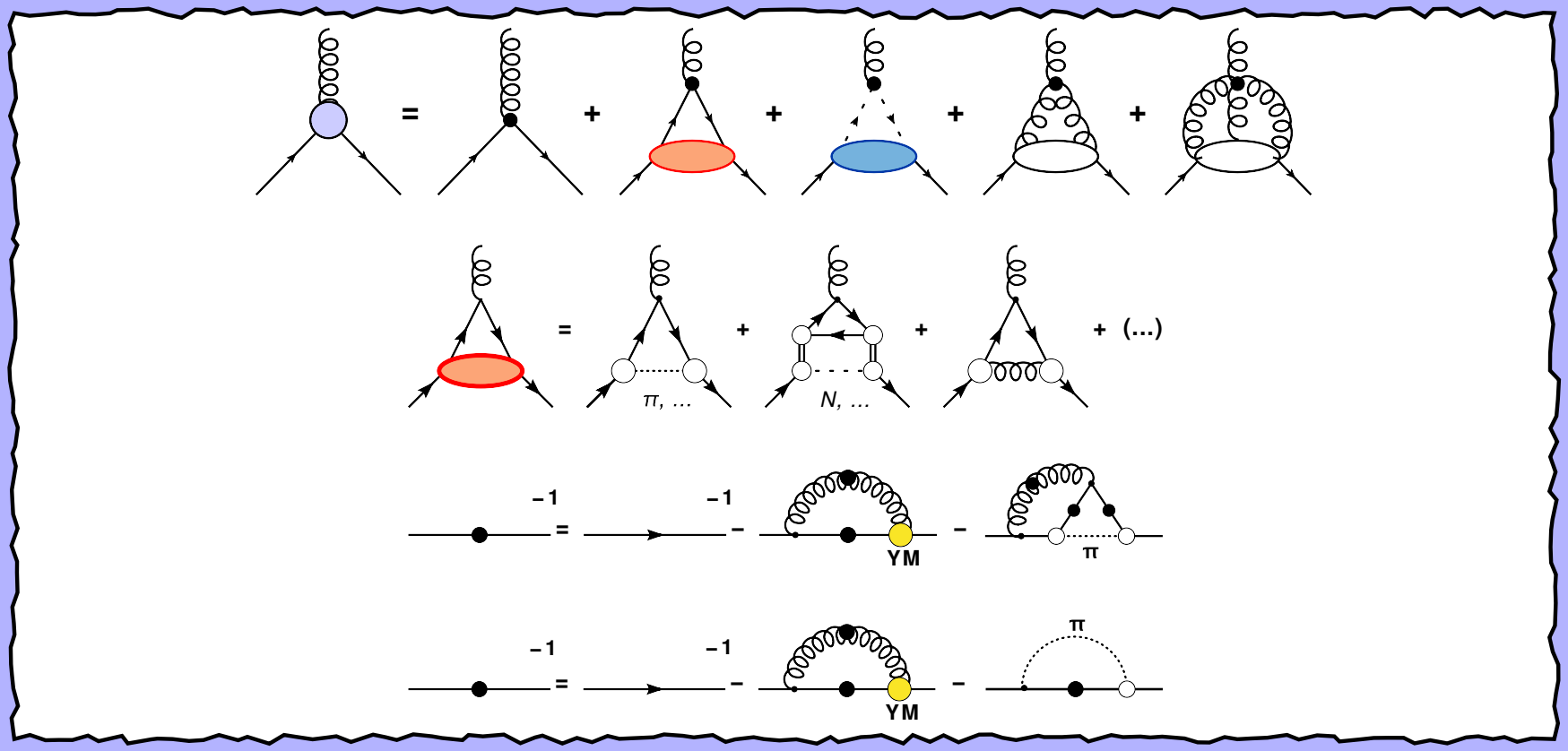


- non-Abelian corrections.
- non-resonant contributions.

Infrared effects + DχSB

[R. Alkofer, C. S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys.324:106-172,2009.]

Quark-Gluon Vertex – Hadronic Contributions I



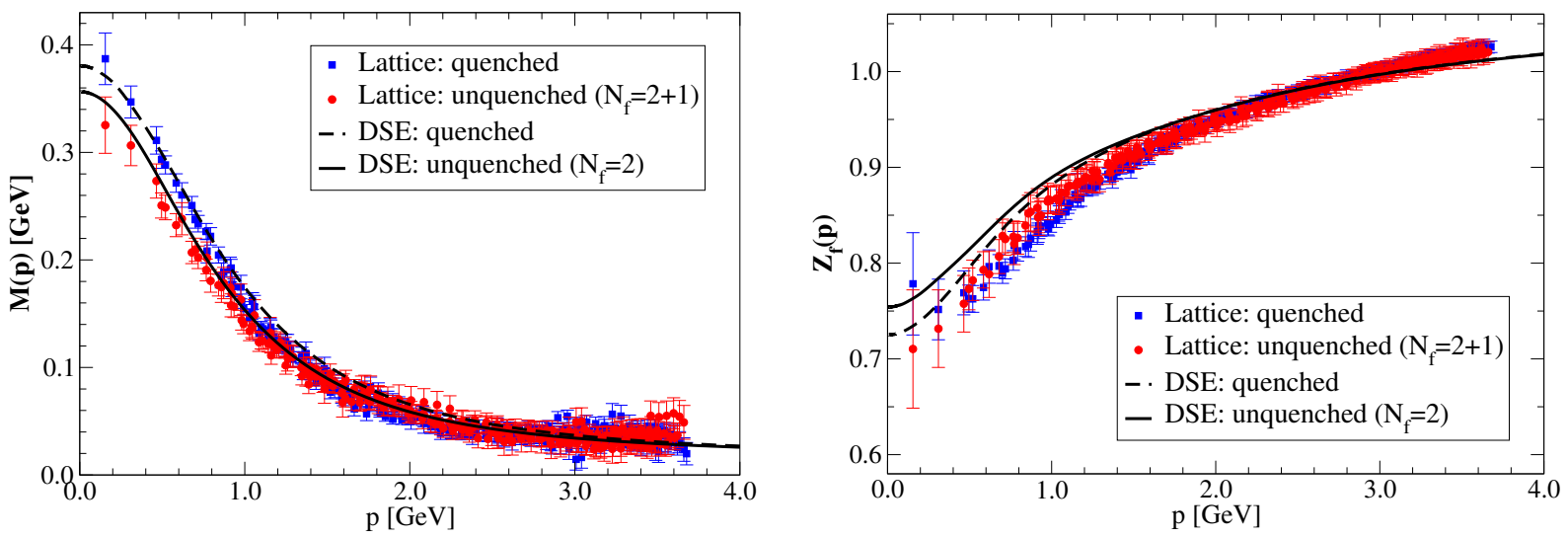
Resonant/Non-resonant components

- pion exchange
- gluon exchange
- . . .

[C. S. Fischer, D. Nickel and J. Wambach, Phys.Rev.D76:094009,2007]

[C. S. Fischer and RW, Phys.Rev.D78:074006,2008]

Quark-Gluon Vertex – Hadronic Contributions II



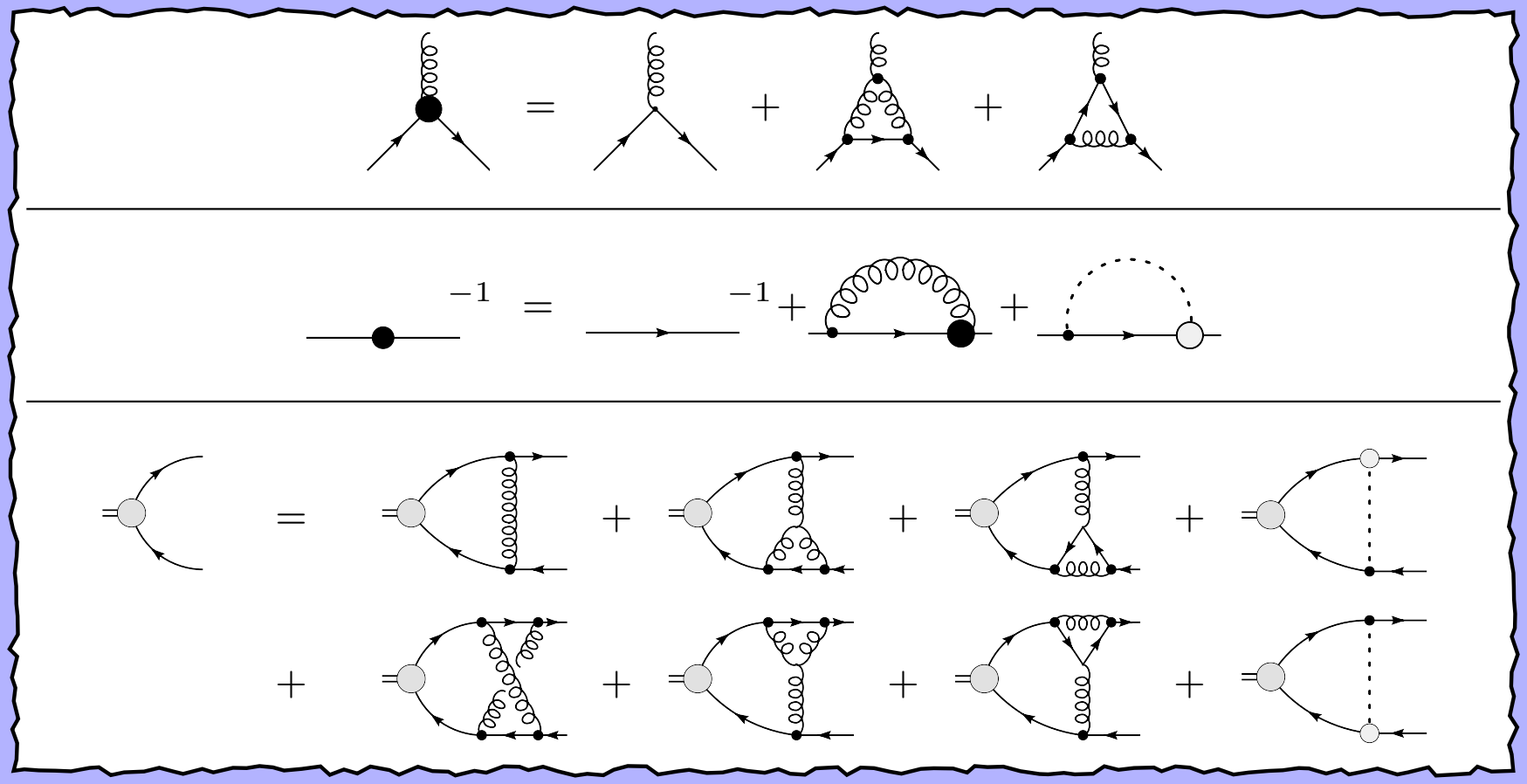
Lattice Comparison

- Impact of back-reaction consistent with quenched vs unquenched lattice† data.

† [P. O. Bowman *et al*, Phys.Rev.D71:054507,2005]

[C. S. Fischer, D. Nickel and RW, Eur.Phys.J.C60:1434-6052,2008]

Beyond Rainbow-Ladder



Symmetry Preserving

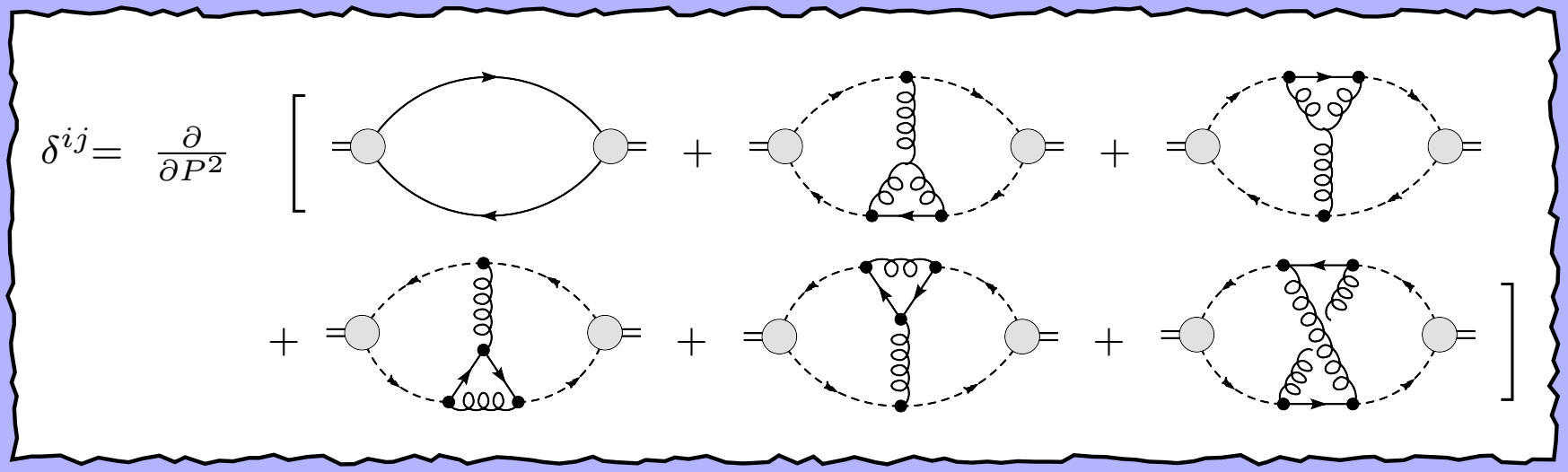
- Cutting rules $\longrightarrow$ AXWTI preserved.

[H. J. Munczek, Phys. Rev. D **52** (1995) 4736]

[A. Bender, C. D. Roberts, L. von Smekal, Phys. Rev. Lett. B **380** (1996) 7]

Normalisation: Leon–Cutkosky

$$\delta^{ij} = \frac{\partial}{\partial P^2}\, tr \int_k \left[\left(\overline{\Gamma}^i_\pi S \Gamma^j_\pi S \right) + \int_q \left([\overline{\chi}^i_\pi]_{sr} K_{tu;rs} [\chi^j_\pi]_{ut} \right) \right]$$



usual approach to determining normalisation condition

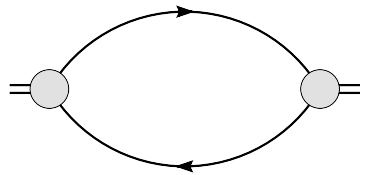
lines	quark propagator.
dashed lines	quark propagator fixed wrt. derivative.
wiggles	gluon propagator.

[R. E. Cutkosky and M. Leon, Phys. Rev. **135** (1964) B1445.]

Normalisation: Nakanishi

$$\left(\frac{d\ln(\lambda)}{dP^2}\right)^{-1} = tr\int_k \bar{\Gamma} S \Gamma S$$

$$\left(\frac{d\ln(\lambda)}{dP^2}\right)^{-1} = \Bigg[$$



$$\Bigg]$$

Where λ is the eigenvalue obtained via $\Gamma = \lambda K \Gamma$.

alternative hidden in the literature

- considerably simpler
- valid for all truncations
- first time applied beyond rainbow-ladder

[N. Nakanishi, Phys. Rev. **138**, B1182 (1965)]

Choose the truncation

Aim *formulate consistent truncation scheme and calculate:*

- quark-gluon vertex – leading corrections + unquenching
- quark propagator
- couple in the Yang-Mills sector:
  - gluon/ghost propagator, three-gluon vertex

Feasibility Study

- effective interaction for gluons
- No momentum trivialisation!
- Solve complicated system of equations
- Gauge impact of diagrammatic contributions

Use a finite width representation of delta-function:
(parameters tuned to reproduce meson observables in Rainbow-Ladder)

[R. Alkofer, P. Watson and H. Weigel, Phys. Rev. D 65 (2002) 094026]

Results

Calculate:

- Propagators and vertices: $\mathbb{C}$ momenta.
- Homogeneous BS amplitude solved

Model	m_π	m_σ	m_ρ	m_{a_1}	m_{b_1}
RL	138	645	758	926	912
NA	142	884	881	1056	973
AB	137	602	734	889	915
AB+NA	142	883	878	1052	972
NA+PI	138	820	805	1040	941
PDG	138	400–1200	776	1230	1230

All masses are in MeV.

[C. S. Fischer and RW, Phys.Rev.Lett.103:122001,2009]

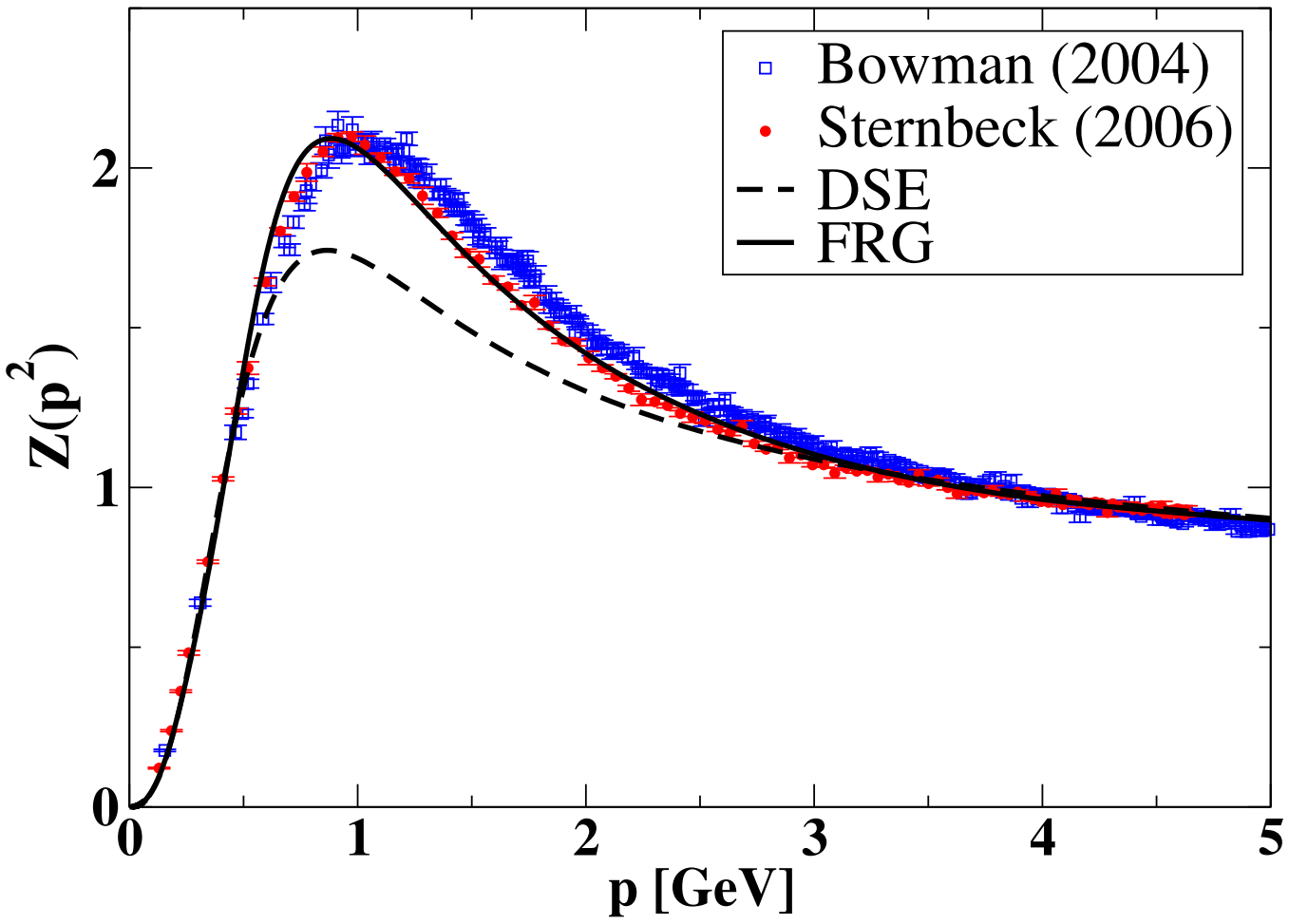
[RW, *proceedings*, arXiv:0912.3494]

RL – Rainbow-Ladder *AB – Abelian correction*

NA – Non-Abelian correction *PI – Resonant Pion contribution.*

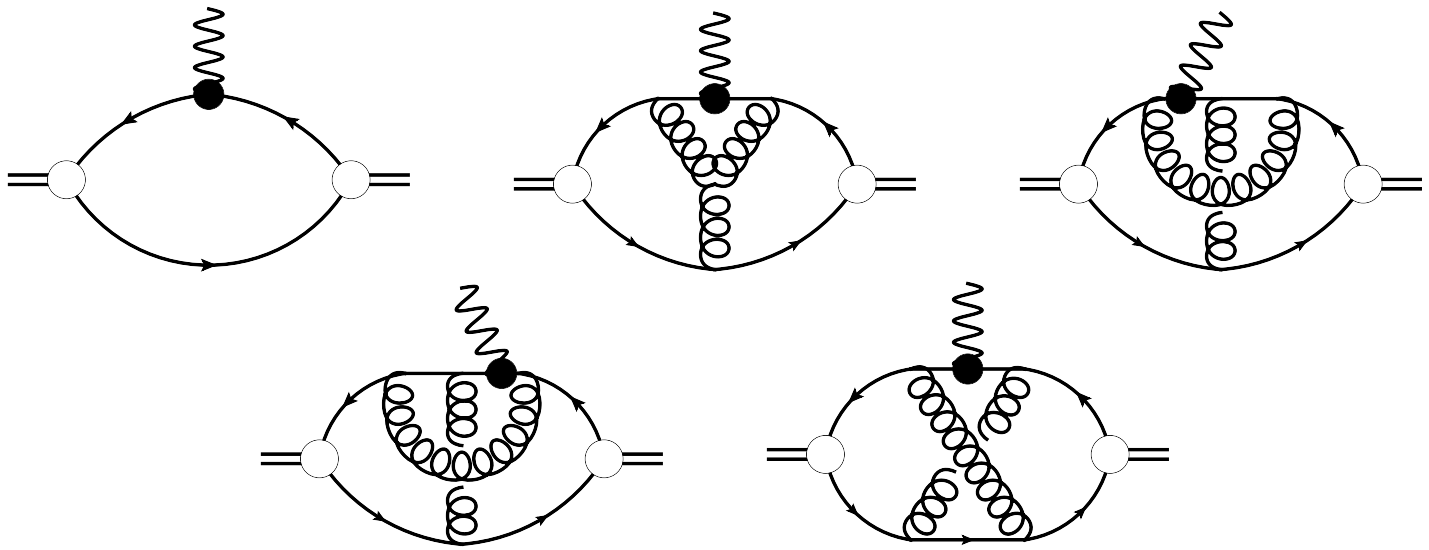
Next steps

Connect with well-known calculated Green's functions



Next steps

Pion electromagnetic form-factor



Beyond Impulse-Approximation

- ✓ Consistent with BSE truncation – must satisfy current conservation
- ✓ Dressed quark-photon vertex
- ✗ Explicitly three-loop

include inputs from gluon DSE, internal quark-gluon vertex, dressed three-gluon vertex.

$\rightarrow$ examine meson spectrum

Conclusions

Summary

- outlined framework for DSE/BSE calculations of QCD
- systematic improvements beyond Rainbow-Ladder
  - leading unquenching effects
  - leading Yang-Mills corrections
  - sub-leading YM corrections
- numerical techniques
- normalisation
- results from feasibility study

Outlook

- Take inputs from DSE solutions – self-consistent truncation of QCD
- Calculate basic meson observables
- Pion electromagnetic form-factor
- ...
- Connect with baryon studies (Eichmann 6C)